MATHEMATICS SPECIALIST

MAWA Year 12 Examination 2019

Calculator-assumed

Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2019**

Question 9 **(4 marks)**

Solution
With the data given, the interval beween data points is $h=2$ hours The trapezoidal rule yields that the integral is $$I=\frac{1}{2}h[f_0+2(f_1+...+f_5)+f_6]$$ Now $1.57+2.87+3.15+2.77+1.53=11.89$ so that $$I=0.24+2(11.89)+0.16=24.18$$ Hence total electricity generated is approximately 24.18 kWh

Mathematical behaviours	Marks
• notes the interval between given data points • writes down the appropriate composite trapezoidal rule • computes the appropriate integral (-1 for one mistake)	1 1 2

Question 10(a) **(2 marks)**

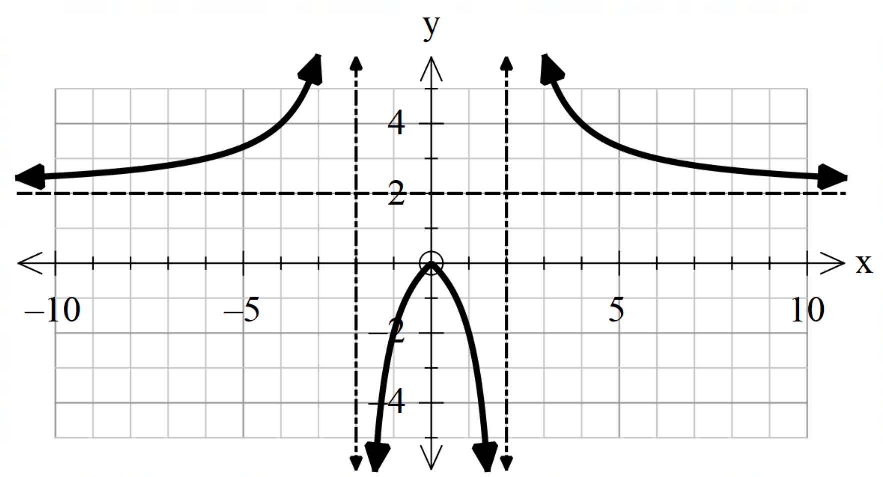
Solution

$y = h(x)$	$y = \dfrac{1}{h(x)}$
Vertical asymptote at $x = 2$	x-intercept at (2,0)
Horizontal asymptote at y = 2	Horizontal asymptote at $y = \dfrac{1}{2}$

Mathematical behaviours	Marks
• states x-intercept correctly	1
• states equation of horizontal asymptote correctly	1

Question 10(b)(i) **(2 marks)**

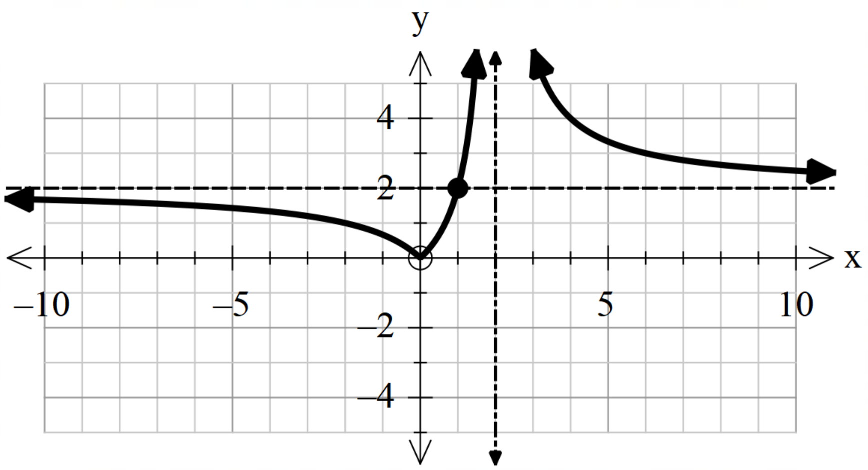
Solution



Mathematical behaviours	Marks
• indicates asymptotes, $y = 2, x = 2, x = -2$ correctly	1
• indicates (0,0) and correct behavior of curve for $-2 < x < 2$, $x \to \pm\infty$, $x \to 2^+, x \to -2^-$	1

Question 10(b)(ii) **(2 marks)**

Solution



Mathematical behaviours	Marks
• indicates asymptote x = 2, (0,0), (1,2) correctly	1
• correct shape of the curve	1

Question 11(a) **(1 mark)**

Solution	
Lines parallel to the x-axis below $y<0$ cut the curve twice $g(x)$ is not a one-to-one function, therefore $g^{-1}(x)$ does not exist	
Mathematical behaviours	Marks
• states the graph of $g(x)$ does not pass the horizontal line test or states $g(x)$ is not a one to one function	1

Question 11(b) **(2 marks)**

Solution	
$R_g = \{y : y \leq 2, y \in \mathbb{R}\}$ $D_f = \{x : x \leq 2, x \in \mathbb{R}\}$ $f \circ g(x)$ exists because $R_g \subseteq D_f$	
Mathematical behaviours	Marks
• states R_g and D_f correctly • states a reason	1 1

Question 11(c) **(1 mark)**

Solution	
$f\left[\dfrac{2x}{(x+2)^2}\right] = \left[\dfrac{2x}{(x+2)^2}\right]^2$ $= \dfrac{4x^2}{(x+2)^4}$, $x \neq -2$	
Mathematical behaviours	Marks
• forms a correct expression for $f \circ g(x)$	1

Question 11(d) **(1 mark)**

Solution	
$R_{gf} = \{y : y \geq 0, y \in \mathbb{R}\}$	
Mathematical behaviours	Marks
• states that $y \geq 0$	1

Question 12(a) **(3 marks)**

Solution	
$xy\dfrac{dy}{dx}=y^2-1$ $\int xy\ dy=\int\left(y^2-1\right)dx$ $\int\dfrac{y}{\left(y^2-1\right)}\ dy=\int\dfrac{1}{x}\ dx$ --------------(1) $\dfrac{1}{2}\int\dfrac{2y}{\left(y^2-1\right)}\ dy=\int\dfrac{1}{x}\ dx$ $\dfrac{1}{2}\ln\left\|y^2-1\right\|=\ln\left\|x\right\|+c$ --------------(2) $\ln\left\|y^2-1\right\|=2\ln\left\|x\right\|+2c$ $\ln\left\|y^2-1\right\|=\ln\left\|k\ x^2\right\|$ i.e. $y^2-1=k\ x^2$ i.e. $y^2=k\ x^2+1$.	
Mathematical behaviours	**Marks**
• separates variables to form statement (1) • integrates correctly to form statement (2) or its equivalent • obtains expression for general solution	1 1 1

Question 12(b) **(2 marks)**

Solution	
$x=1.y=0\Rightarrow y^2=k\ x^2+1$ $0=k\ (1)^2+1$ $k=-1$ i.e $y^2=-x^2+1$ or $x^2+y^2=1$ is a circle with radius of 1 unit and centre at (0,0)	
Mathematical behaviours	**Marks**
• obtains correct expression for equation of circle • states radius and coordinates of centre correctly	1 1

Question 13(a) **(3 marks)**

Solution
$x^2+1+e^{x+y}=(2y-1)^2$ $\frac{d}{dx}\left(x^2+1+e^{x+y}\right)=\frac{d}{dx}\left[(2y-1)^2\right]$ $2x+e^{x+y}\left(1+\frac{dy}{dx}\right)=2(2y-1).2.\frac{dy}{dx}$ $2x+e^{x+y}+e^{x+y}\frac{dy}{dx}=4(2y-1)\frac{dy}{dx}$ $2x+e^{x+y}=\left[4(2y-1)-e^{x+y}\right]\frac{dy}{dx}$ $\frac{2x+e^{x+y}}{\left[4(2y-1)-e^{x+y}\right]}=\frac{dy}{dx}$ i.e $\frac{dy}{dx}=\frac{2x+e^{x+y}}{4(2y-1)-e^{x+y}}$

Mathematical behaviours	Marks
• differentiates e^{x+y} implicitly correctly	1
• differentiates $(2y-1)^2$ with respect to x correctly	1
• obtains the required expression for $\frac{dy}{dx}$	1

Question 13(b) **(5 marks)**

Solution
$a=4v^2$ $\frac{dv}{dt}=4v^2$ $\frac{dx}{dt}\times\frac{dv}{dx}=4v^2$ $v\times\frac{dv}{dx}=4v^2$ ---------------------(1) $\frac{dv}{dx}=4v$ $\int\frac{dv}{4v}=\int dx$ $\frac{1}{4}\int\frac{1}{v}dv=\int dx$ ------------------(2) $\frac{1}{4}\ln\lvert v\rvert=x+c$ -------------------(3) $x=2, v=e^5\Rightarrow\frac{1}{4}\ln\lvert e^5\rvert=2+c$ $c=-\frac{3}{4}$ $\therefore\frac{1}{4}\ln\lvert v\rvert=x-\frac{3}{4}$ $x=1, v=?\Rightarrow\frac{1}{4}\ln\lvert v\rvert=1-\frac{3}{4}$ $\frac{1}{4}\ln\lvert v\rvert=\frac{1}{4}$ $\ln\lvert v\rvert=1$ $\therefore v=e$

Mathematical behaviours	Marks
• identifies $a=\frac{dv}{dt}$ and uses $\frac{dv}{dt}=v\frac{dv}{dx}$ to form statement (1)	1
• simplifies and separates variables to form statement (2) or its equivalent	1
• anti-differentiates to obtain statement (3) or its equivalent	1
• use $x=2, v=e^5$ to determine the constant of integration correctly	1
• determines the velocity correctly when $x=1$	1

Question 14(a) **(3 marks)**

Solution
If $z^5 = -1$ then $z^5 = \cos(2k+1)\pi + i\sin(2k+1)\pi$ for $k \in \mathbb{Z}$ By de Moivre's theorem then $z = \exp\left(i\vartheta\right)$ where $\vartheta = \frac{(2k+1)\pi}{5}$ Hence the distinct roots are $z = \exp\left(i\vartheta\right)$ where $\vartheta = \frac{(2k+1)\pi}{5}$, $k = 0...4$ For arguments in the range given this is equivalent to $z = \exp(i\vartheta)$, $\vartheta = \pm\frac{3\pi}{5}, \pm\frac{\pi}{5}, \pi$

Mathematical behaviours	Marks
• writes -1 in appropriate polar form	1
• applies de Moivre's theorem correctly	1
• restricts the arguments of the solutions to the specified range	1

Question 14(b) **(3 marks)**

Solution
We note that $(2i)^5 = 32i^5 = 32i$ Hence $$(z-1)^5 + (2i)^5 = 0 \quad \Rightarrow \quad \left(\frac{z-1}{2i}\right)^5 + 1 = 0$$ From part (a) we conclude that $$\frac{z-1}{2i} = \exp(i\vartheta) \ \Rightarrow \ z = 1 + 2i\exp(i\vartheta)$$ with the arguments ϑ as given in part (a)

Mathematical behaviours	Marks
• realises that $32i = (2i)^5$	1
• divides equation through by $(2i)^5$ thereby reducing the equation to the form in (a)	1
• deduces the five roots of the modified equation	1

Question 15(a) **(2 marks)**

Solution	
$E(\overline{X}) = E(X) = 0.5$ $Var(\bar{X}) = \frac{Var(X)}{n} = \frac{1}{24} \cong 0.042$	
Mathematical behaviours	**Marks**
• obtains correct answer for $E(\overline{X})$ • obtains correct answer for $Var(\bar{X})$	1 1

Question 15(b) **(2 marks)**

Solution	
$\bar{X}$ is normally distributed (*) $P(\bar{X} \leq 0.25) = P\left(Z \leq \frac{0.25 - 0.5}{\sqrt{\frac{1}{24}}}\right) = P\left(Z \leq \frac{0.25 - 0.5}{\sqrt{\frac{1}{24}}}\right) = P(Z \leq -1.224) = 0.19$	
Mathematical behaviours	**Marks**
• uses normality • obtains correct answer	1 1

Question 15(c) **(2 marks)**

Solution	
$\bar{X} \leq 0.25 \Leftrightarrow \frac{x+y}{2} \leq 0.25 \Leftrightarrow x + y \leq 0.5$ So the required probability equals the area of the shaded triangle (*) i.e. 0.125	
Mathematical behaviours	**Marks**
• obtains equality (*) • obtains correct answer	1 1

Question 15(d) **(3 marks)**

Solution	
$\bar{X} \leq 0.25 \Leftrightarrow \frac{x+y}{2} \leq 0.25 \Leftrightarrow x + y \leq 0.5$ Because the sample size is large enough, the distribution of $\bar{X}$ is approximately normal. (*) The mean is 0.5 and the variance is $\frac{1}{12n} = \frac{1}{120} \cong 0.00833$ (**) So $P(\bar{X} \leq 0.25) \cong 0.003$	
Mathematical behaviours	**Marks**
• uses normal approximation • uses correct variance (**) • obtains correct answer	1 1 1

Question 16(a) **(2 marks)**

Solution
If $u=\cos x$ then $u'(x)=-\sin x$ and $$\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\,dx=\int_1^{-1}\frac{(-du)}{1+u^2}=\int_{-1}^{1}\frac{du}{1+u^2}=\left[\arctan u\right]_{-1}^{1}=\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)=\frac{\pi}{2}$$

Mathematical behaviours	Marks
• uses the given change of variable to write integral in terms of u • integrates correctly	1 1

Question 16(b) **(3 marks)**

Solution
If $v=a-x$ then $dv=-dx$ and so $$\int_0^{a} f(x)\,dx=\int_a^{0} f(a-v)(-dv)=\int_0^{a} f(a-v)\,dv$$ Since v is a dummy variable this integral is equal to $\int_0^{a} f(a-x)\,dx$ as required

Mathematical behaviours	Marks
• writes the integral in terms of the variable v • realises that the minus sign in the derivative can be accounted for by interchanging limits	1 1

Question 16(c) **(5 marks)**

Solution
If $I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}\,dx$ then using the result of part (b) with $f(x)=\frac{x\sin x}{1+\cos^2 x}$ gives that $$I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$$ Now $\sin(\pi-x)=\sin x$ and $\cos(\pi-x)=-\cos x$ so that $$I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx=\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\,dx-I$$ Thus using the result of part (a) $$I=\pi\left(\frac{\pi}{2}\right)-I \quad\Rightarrow\quad 2I=\frac{\pi^2}{2} \quad\Rightarrow I=\frac{\pi^2}{4}$$

Mathematical behaviours	Marks
• writes down the form of the integral using the structure of part (b)	1
• establishes the forms $\sin(\pi-x)=\sin x$ and $\cos(\pi-x)=-\cos x$	1
• simplifies the integrand	1
• realises that the integral I is now part of the integral in part (a) and I itself	1
• combines the previous results to deduce the result	1

Question 17(a) **(4 marks)**

Solution
If $z=1+2i$ then $z^2=-3+4i$, $z^3=(-3+4i)(1+2i)=-11-2i$ and $z^4=(-11-2i)(1+2i)=-7-24i$ Since $P(z)\equiv z^4-8z^3+42z^2+\alpha z+\beta$, $P(1+2i)=-7-24i-8(-11-2i)+42(-3+4i)+\alpha(1+2i)+\beta=0$ Imaginary parts give $-24+16+168+2\alpha=0\Rightarrow 2\alpha=-160\Rightarrow\alpha=-80$ and real parts give $-7+88-126-80+\beta=0\Rightarrow\beta=125$

Mathematical behaviours	Marks
• computes the values of z^2, z^3 and z^4	1
• substitutes into expression for $P(1+2i)$ and equates to zero	1
• compares imaginary parts to deduce α	1
• compares real parts to deduce β	1

Question 17(b) **(3 marks)**

Solution
As $P(z)$ has real coefficients, if $P(1+2i)=0$ then also $P(1-2i)=0$ Then $(z-1-2i)(z-1+2i)=(z^2-2z+5)$ is a factor of $P(z)$ By long division $P(z)=z^4-8z^3+42z^2-80z+125=(z^2-2z+5)(z^2-6z+25)$

Mathematical behaviours	Marks
• realises that $1-2i$ must also be a root of $P(z)=0$	1
• deduces a quadratic factor of the quartic	1
• deduces the other quadratic using long division	1

Question 18(a) **(2 marks)**

Solution	
$E = z_\alpha \frac{\sigma}{\sqrt{n}}$, i.e. $0.25 = 1.96\frac{3.4}{\sqrt{n}}$ (*) Solving for n gives $n \cong 710.54$ So the sample size should be at least 711	
Mathematical behaviours	Marks
• obtains equation (*) • obtains correct answer	1 1

Question 18(b) **(3 marks)**

Solution	
Confidence interval is $\bar{X} - E \leq \mu \leq \bar{X} + E$, where $E = z_\alpha \frac{\sigma}{\sqrt{n}}$, i.e. $16.2 - 1.96\frac{3.4}{\sqrt{127}} \leq \mu \leq 16.2 + 1.96\frac{3.4}{\sqrt{127}}$ i.e. $15.61 \leq \mu \leq 16.79$	
Mathematical behaviours	Marks
• uses correct formula (*) • obtains correct centre of CI (implicitly at least) • obtains width of CI (implicitly at least)	1 1 1

Question 18(c) **(2 marks)**

Solution	
The evidence provided by the sample suggests that there has been a change in TV viewing time, but it is hardly compelling, since the old mean (15.7) lies inside the confidence interval.	
Mathematical behaviours	Marks
• obtains correct conclusion • provides a valid reason	1 1

Question 19(a) **(1 mark)**

Solution	
$\frac{dN}{dt}=\frac{1}{5000}\times 100\times (400)$ $\frac{dN}{dt}=8$	
Mathematical behaviours	Marks
• substitutes $N=100$ into equation to solve for $\frac{dN}{dt}$ correctly	1

Question 19(b) **(1 mark)**

Solution	
As $N\to 500,\ \frac{dN}{dt}\to 0$ i.e. as the number of nesting pairs of black terns approaches 500, their rate of increase approaches zero	
Mathematical behaviours	Marks
• states $\frac{dN}{dt}$ tends to zero and gives interpretation correctly	1

Question 19(c) **(3 marks)**

Solution
$\frac{dt}{dN}=\frac{10}{N}+\frac{10}{500-N}$ $\int dt=\int\left(\frac{10}{N}+\frac{10}{500-N}\right)dN$ $t=10\ln\lvert N\rvert-10\ln\lvert 500-N\rvert+c$ -------------------- (1) $t=10\ln\left\lvert\frac{N}{500-N}\right\rvert+c$ $t=0, N=100\Rightarrow\ 0=10\ln\left\lvert\frac{100}{500-100}\right\rvert+c$ $0=10\ln\left\lvert\frac{1}{4}\right\rvert+c$ $c=10\ln 4$ i.e $t=10\ln\left\lvert\frac{N}{500-N}\right\rvert+10\ln 4$ $\therefore\ t=10\ln\left\lvert\frac{4N}{500-N}\right\rvert$

Mathematical behaviours	Marks
• integrates correctly to form statement (1) or its equivalent • use the condition $t = 0, N = 100$ to determine the correct constant of integration • obtains the required expression for t	1 1 1

Question 19(d) **(2 marks)**

Solution	
$t = 10\ln\left\|\frac{4N}{500-N}\right\|$ $\frac{t}{10} = \log_e\left\|\frac{4N}{500-N}\right\|$ $e^{\frac{t}{10}} = \frac{4N}{500-N}$ $500e^{\frac{t}{10}} - e^{\frac{t}{10}}N = 4N$ $500e^{\frac{t}{10}} = 4N + e^{\frac{t}{10}}N$ $500e^{\frac{t}{10}} = N\left(4 + e^{\frac{t}{10}}\right)$ $N = \frac{500e^{\frac{t}{10}}}{4+e^{\frac{t}{10}}} = \frac{500}{4e^{-\frac{t}{10}}+1}$.	
Mathematical behaviours	**Marks**
• expresses $\frac{4N}{500-N}$ in terms of an exponential function • rearranges and obtains an expression for *N* correctly	1 1

Question 19(e) **(2 marks)**

Solution	
When $t = 18$ then $N \approx 300.99$ or $N = 301$ to the nearest integer	
Mathematical behaviours	**Marks**
• substitutes value of t in the appropriate equation • solves correctly for *N* to the nearest integer	1 1

Question 20(a) **(2 marks)**

Solution
The given curve cuts the x-axis at $x = 0, 2$ Thus $$A = \int_0^2 kx(2-x)\,dx = k\int_0^2 (2x - x^2)\,dx = k\left[x^2 - \frac{1}{3}x^3\right]_0^2 = k\left(4 - \frac{8}{3}\right) = \frac{4k}{3}$$

Mathematical behaviours	Marks
• identifies the limits of the integration • deduces the correct value of the area	1 1

Question 20(b) **(2 marks)**

Solution
Now $$V_1 = \pi\int_0^2 y^2\,dx = \pi k^2\int_0^2 (4x^2 - 4x^3 + x^4)\,dx = \pi k^2\left[\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5\right]_0^2 = \pi k^2\left[32\left(\frac{1}{3} + \frac{1}{5}\right) - 16\right]$$ $$= 16\pi k^2\left[\frac{16}{15} - 1\right] = \frac{16}{15}\pi k^2$$

Mathematical behaviours	Marks
• integrates correctly • simplifies to derive the correct answer	1 1

Question 20(c) **(6 marks)**

Solution
For each value of $y \in [0,k]$ there are two values x_1 and $x_2\ (> x_1)$ for which $kx(2-x) = y$ Now required volume V_2 obtained by rotating area A about the y-axis is $$V_2 = \pi\int_0^k \left(x_2^2 - x_1^2\right)dy$$ Now x_1 and x_2 are the roots of $x^2 - 2x + (y/k) = 0$ so $x_{1,2} = 1 \pm \sqrt{1-(y/k)}$ Then we deduce that $x_2^2 - x_1^2 = (x_2 - x_1)(x_2 + x_1) = 4\sqrt{1-(y/k)}$. Hence $$V_2 = 4\pi\int_0^k \sqrt{1-\frac{y}{k}}\,dy$$ and if $u = 1-(y/k)$ then $$V_2 = \pi\int_1^0 \sqrt{u}\,(-k)du = k\pi\left[\frac{2}{3}u^{3/2}\right]_0^1 = \frac{2k\pi}{3}$$ Then, if $V_1 = V_2$ so $$\frac{16}{15}k^2 = \frac{2}{3}k \Rightarrow k = \frac{5}{8}$$

Mathematical behaviours	Marks
• notes that for each value of y in range there are two corresponding x • derives expressions for x_1 and x_2 in terms of y • writes down the integral for V_2 in terms of y • uses an appropriate substitution to evaluate the integral • equates with V_1 • deduces the required value for k	1 1 1 1 1 1

Question 21(a) **(2 marks)**

Solution	
$\boldsymbol{r}_1(t) = 5t\boldsymbol{i} + 4t\boldsymbol{j} + 16\boldsymbol{k}$ So $\boldsymbol{v_1}(t) = 5\boldsymbol{i} + 4\boldsymbol{j}$ (*) and so the speed is given by $\|\boldsymbol{v_1}(t)\| = \sqrt{5^2 + \sqrt{4^2}} = \sqrt{41} \cong 6.40\ cms^{-1}$	
Mathematical behaviours	Marks
• differentiates correctly (*) • obtains correct answer	1 1

Question 21(b) **(3 marks)**

Solution	
$\boldsymbol{v_2}(t) = \int_0^t \boldsymbol{a}(u)du + \boldsymbol{v_2}(0) = \int_0^t -2\boldsymbol{k}du + \boldsymbol{i} + 2\boldsymbol{j} + 8\boldsymbol{k} = \boldsymbol{i} + 2\boldsymbol{j} + (8-2t)\boldsymbol{k}$ (*) and $\boldsymbol{r_2}(t) = \int_0^t \boldsymbol{v}(u)du + \boldsymbol{r_2}(0) = \int_0^t(\boldsymbol{i} + 2\boldsymbol{j} + (8-2u)\boldsymbol{k})du + 6\boldsymbol{i}$ $= (t+6)\boldsymbol{i} + 2t\boldsymbol{j} + (8t - t^2)\boldsymbol{k}$ (**) So the height at time t is $8t - t^2$ The maximum of $8t - t^2$ occurs when $t = 4$ and is 16. So the maximum height is 16 cm.	
Mathematical behaviours	Marks
• solves for $\boldsymbol{r_2}(t)$ (**) • find value of t which maximizes $8t - t^2$ • obtains correct answer	1 1 1

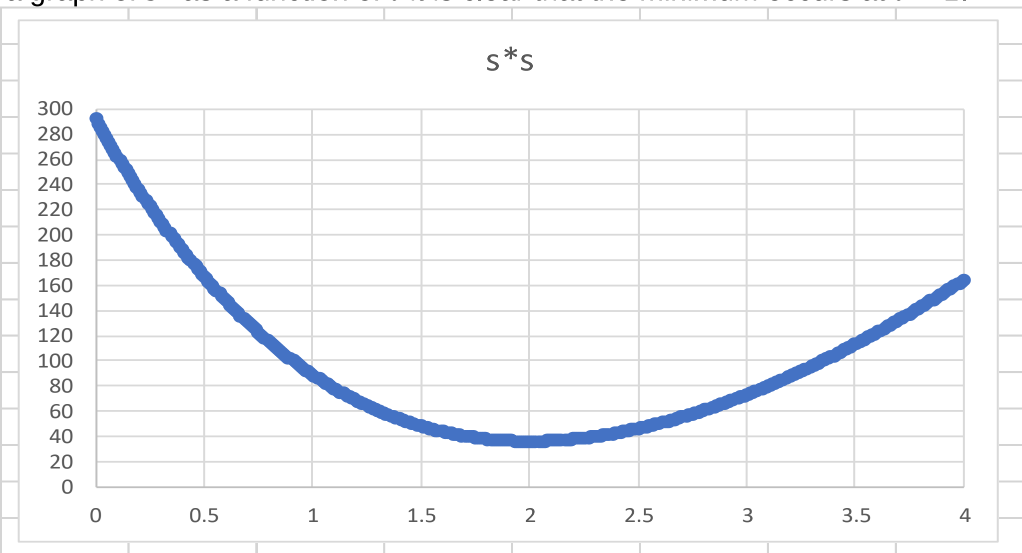
Question 21(c) **(3 marks)**

Solution	
The flight paths intersect if $\boldsymbol{r_1}(t) = \boldsymbol{r_2}(t')$ for some values of t and t'. $\boldsymbol{r_1}(t) = \boldsymbol{r_2}(t')$ implies $8t' - t'^2 = 16$, i.e. $t' = 4$. $\boldsymbol{r_2}(4) = 10\boldsymbol{i} + 8\boldsymbol{j} + 16\boldsymbol{k}$ Now $\boldsymbol{r_1}(2) = 10\boldsymbol{i} + 8\boldsymbol{j} + 16\boldsymbol{k}$ So the paths intersect at the point with coordinates (10,8,16) Since the particles are at the intersection point at different times, there is no collision.	
Mathematical behaviours	Marks
• uses $\boldsymbol{r_1}(t) = \boldsymbol{r_2}(t')$ (*) • shows that the paths intersect • shows that there is no collision	1 1 1

Question 21(d) **(2 marks)**

Solution	
$\boldsymbol{r_1}(t) - \boldsymbol{r_2}(t) = (4t-6)\boldsymbol{i} + (2t)\boldsymbol{j} + (16-8t+t^2)\boldsymbol{k}$ (*) So $d^2 = (4t-6)^2 + (2t)^2 + (16-8t+t^2)^2 = 4(2t-3)^2 + 4t^2 + (t-4)^4$	
Mathematical behaviours	**Marks**
• finds $\boldsymbol{r_1}(t) - \boldsymbol{r_2}(t)$ (*)	1
• evaluates $d^2 = \lVert \boldsymbol{r_1}(t) - \boldsymbol{r_2}(t) \rVert^2$	1

Question 21(e) **(2 marks)**

Solution	
Using a graph of s^2 as a function of t it is clear that the minimum occurs at $t = 2$.	
	
(This can be checked using a derivative, but this is not required: $\frac{d(s^2)}{dt} = 4t^3 - 48t^2 + 232t - 304 = 0$ when $t = 2$. So $t = 2$ is indeed a critical point.)	
Mathematical behaviours	**Marks**
• shows a sketch graph or calculates a derivative	1
• answers correctly	1

Question 21(f) **(3 marks)**

Solution	
$\boldsymbol{r_1}(2) - \boldsymbol{r_2}(2) = 2\boldsymbol{i} + 4\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{v_1}(2) - \boldsymbol{v_2}(2) = 4\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$ So $(\boldsymbol{r_1}(2) - \boldsymbol{r_2}(2)).(\boldsymbol{v_1}(2) - \boldsymbol{v_2}(2)) = 2 \times 4 + 4 \times 2 + 4 \times (-4) = 0$ (*) Since the dot product is 0 the vectors are perpendicular.	
Mathematical behaviours	**Marks**
• evaluates $\boldsymbol{r_1}(2) - \boldsymbol{r_2}(2)$ and $\boldsymbol{v_1}(2) - \boldsymbol{v_2}(2)$	1
• evaluates dot product (*)	1
• gives a valid reason	1